Three-dimensional AEM inversion considering IP effect using Octree mesh

Jian Chen

The EEM Team for Hydro & eXploration, Department of Earth Sciences "Ardito Desio", University of Milano, Milano (Italy).
jian.chen@unimi.it

Bo Zhang

Institute of Earth exploration, Science and Technology, Jilin University, Changchun (China).
em_zhangbo@163.com

Gianluca Fiandaca

The EEM Team for Hydro & eXploration, Department of Earth Sciences "Ardito Desio", University of Milano, Milano (Italy).
gianluca.fiandaca@unimi.it

SUMMARY

Modelling Induced polarization effects in airborne electromagnetic (AEM) data is becoming a standard tool in mineral exploration, but the industry standard is still based on one-dimensional (1D) forward and Jacobian modelling. We have developed a three-dimensional (3D) vector finite element electromagnetic forward and inversion method considering IP effects within the EEMverter modelling platform. The computations are carried out in frequency domain, and then time-transformed in time domain through a Hankel transform. This allows to model any parameterization of the IP phenomenon, from the simple constant phase angle model to a full debye decomposition. Furthermore, 3D forward modeling mesh and inversion mesh are constructed separately: an Octree mesh is designed for efficient spatial segmentation for each sounding, while the inversion parameters are defined on a structured model mesh linked to the forward meshes through interpolation.

Key words: Induced polarization, airborne electromagnetic, three-dimensional, finite element, EEMverter.

INTRODUCTION

Airborne electromagnetic (AEM) exploration technology, recognized for its efficiency, flexibility, and indifference to complex terrains, has been extensively applied in hydrogeological mapping, geothermal exploration, and energy resource surveys (Finn et al., 2022). Due to the typically large-scale datasets collected via AEM, employing inversion methods based on one-dimensional (1D) forward operators remains a conventional and mainstream strategy for data interpretation. In geological settings where the terrain is flat and the subsurface media are approximately layered, 1D inversion can provide relatively accurate interpretations. However, in regions where the terrain is rugged and/or the distribution of subsurface media varies significantly in different directions, such as mineral deposits, 1D forward modeling is no longer sufficient and three-dimensional (3D) inversion is required for proper interpretation.

Current 3D forward modeling strategies primarily include finite difference (FD), finite element (FE), finite volume (FV), and integral equation (IE) methods. Among these, the FD method is limited to structured meshes for numerical simulations, while the FE and FV methods can utilize unstructured meshes, allowing for easier handling of irregular physical property boundaries. Both FE and FV schemes offer comparable accuracy and computational resource requirements, and the FE approach is generally more precise but also more costly (Spitzer 2024). The IE method only requires discretization of the domain of inversion, which has an advantage in calculation speed. However, the need to solve Green's equation limits the ability of the IE method to simulate complex geoelectric models. The FE method partitions the computational space into independent small elements and defines shape functions at nodes or edges of each element. These shape functions enable approximations of the electromagnetic field distribution within the elements using the fields at the nodes or edges. In nodal FE approaches, where shape functions are exclusively distributed at nodes, maintaining the tangential continuity of the electromagnetic fields can be challenging, thus it is typically employed in 2D forward modeling. In contrast, the vector FE define shape functions along the edges of each element, naturally ensuring the tangential continuity of electromagnetic fields and meeting the condition of zero divergence, which aids in eliminating spurious solutions (Yoshimura and Oshiman 2002).
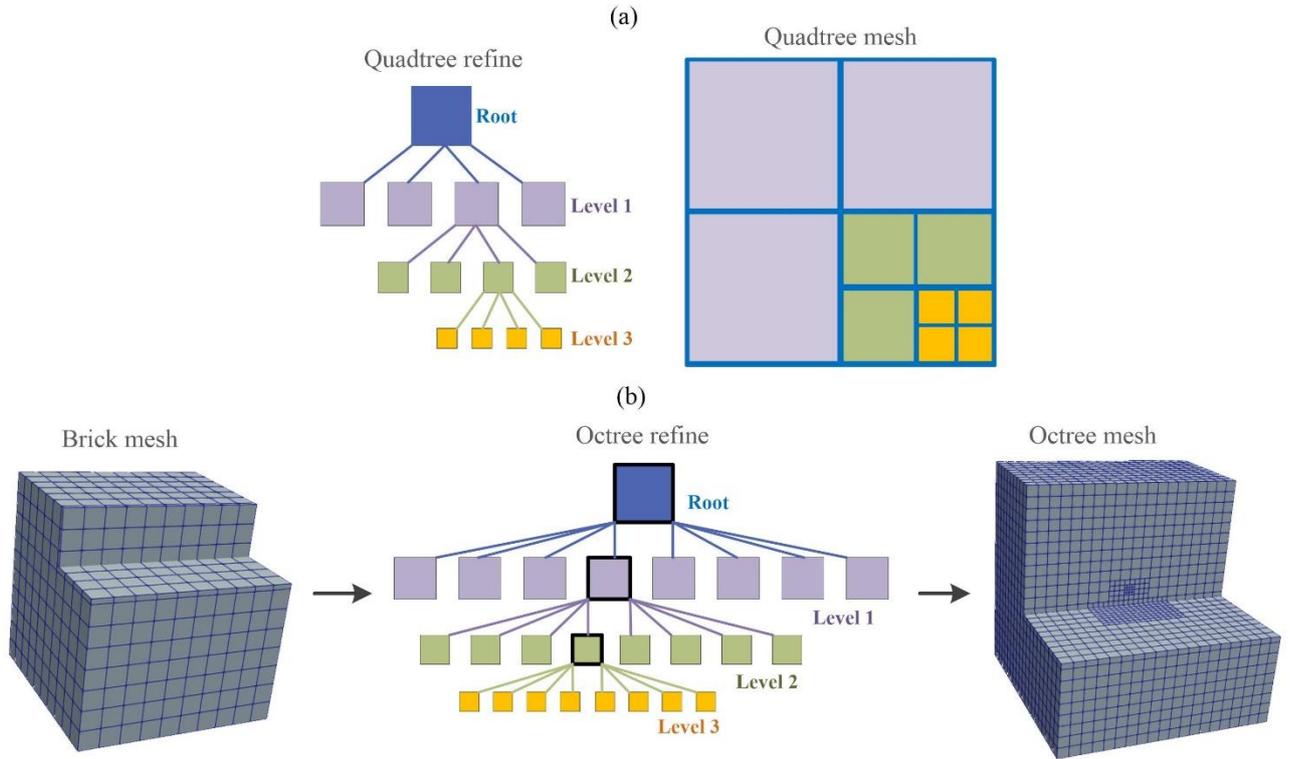
To meet the demand for detailed interpretation of airborne electromagnetic data for mineral resources, this study employs the vector finite element method, which is known for its high flexibility and computational accuracy, to perform 3D AEM forward modelling and inversion. The main features include: 1) the use of Octree meshes to accelerate the meshing process and allow further mesh refinement during inversion iterations, 2) calculation of complex resistivity responses in the frequency domain, enabling easy simulation of any parameterized model of IP, and 3) the independence of the forward modeling mesh from the inversion model mesh, requiring the calculation of the Jacobian matrix only in the footprint area of the forward modeling mesh during inversion iterations.

Method and Theory

In our approach, independent meshes were designed for both the forward modelings and inversion of AEM data through spatial mesh decoupling designs, with inversion parameters defined on the model mesh. This model mesh is distinct from the forward mesh, with its values derived from interpolation of parameters from the forward mesh based on spatial distances. Figure 1 shows a schematic diagram of the Octree grid, where we only need to refine the central footprint area. In our electromagnetic numerical simulation approach, we have adopted a primary-secondary field separation technique. Unlike the traditional total field computation strategy, this method does not require dense meshing near the source and has a smaller footprint area (Zhang et al., 2021), conducive to the substantial conservation of computational resources. By separating the primary field $\mathbf{E}^p$ and the secondary field $\mathbf{E}^s$, the background conductivity σ^p and the abnormal conductivity σ^s, and applying Dirichlet boundary conditions, the frequency domain electromagnetic double-curl partial differential equation is solved based on the vector finite element idea:

$$\int_{\Omega} (\nabla \times \mathbf{N} \cdot \nabla \times \mathbf{E}^s - i\omega\mu(\sigma^s + \sigma^p)\mathbf{N} \cdot \mathbf{E}^s)dV = \int_{\Omega}(i\omega\mu\sigma^s\mathbf{N} \cdot \mathbf{E}^p)dV\,,$$
$$E_s|_{\Omega} = 0. \qquad (1)$$

Figure 1. Diagram of Octree mesh. (a) Quadtree mesh and representative trees; (b) Octree mesh refinement.

Figure 2 demonstrates the accuracy verification of 3D FD numerical simulation under different frequencies in a uniform half-space. Compared to the results calculated from 1D analytical solutions, the error of 3D forward modeling based on Octree mesh is consistently less than 3%, indicating excellent computational precision. Computing the Jacobian matrix is critical for inversion. Figure 3 displays the comparison between the Jacobian matrix values from 3D forward modeling and the values derived from 1D analytical solutions, revealing good consistency between them. The FD computation are then transformed in time domain through a fast Hankel transform. The choice of solving the differential equations in frequency domain, instead of the time stepping approach, allows to easily model induced polarization (IP). The 3D Jacobian matrix is calculated by summing the forward modeling mesh footprint area and subsequently interpolating it into the inversion model mesh. This strategy allows us to easily compute the Jacobian matrix of multi-model parameters. The Frequency-Domain (FD) forward and Jacobian computation are transformed to Time-Domain (TD) through a Hankel Transform, and the AEM-IP inversion is carried out through a Levenberg-Marquardt linearized approach in EEMverter (Fiandaca et al., 2024), a new inversion tool for Electric and Electromagnetic data with focus on Induced Polarization.

Figure 4 presents a comparison between 3D and 1D TD Jacobian matrices, taking into account IP effects. The IP effects are modeled using the maximum phase angle (MPA) re-parameterization model, which serves to mitigate the correlation among polarization parameters and enhances the reliability of subsequent inversion interpretations. The values of the 3D Jacobian matrix for each polarization parameter correspond well with those of the 1D Jacobian matrix. Each forward mesh only needs to be locally refined in the underground sensitive areas covered by the transmitter and receiver.
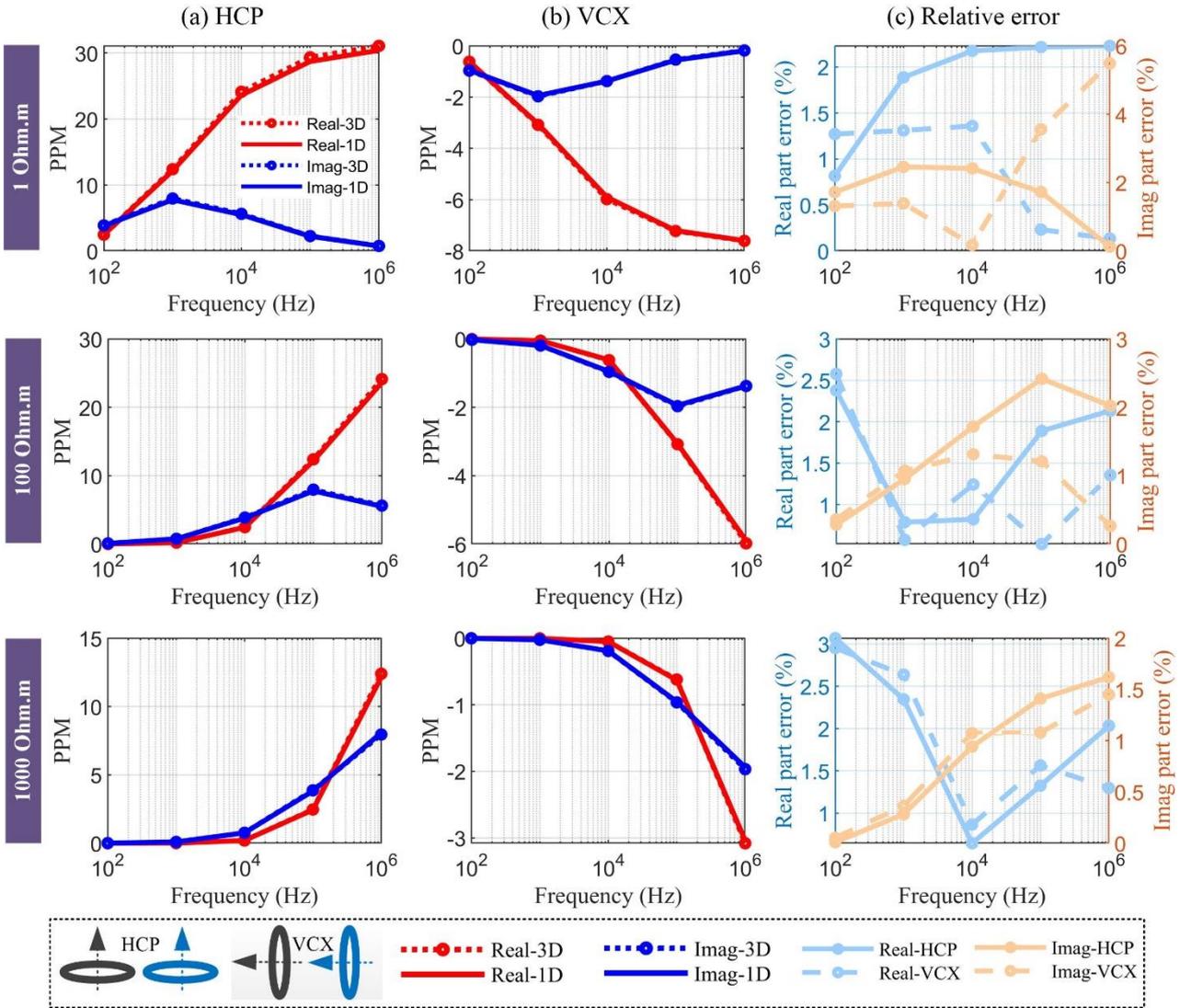
Figure 2. Comparison of calculation accuracy of three-dimensional vector finite element frequency domain electromagnetic forward modeling based on Octree mesh. The first row of graphs shows the comparison of PPM response values for HCP configuration, the second row shows the PPM response values for VCX configuration, and the third row shows the relative percentage error. (a) Half-space resistivity is 1 Ohmm; (b) Half-space resistivity is 100 Ohmm; (c) Half-space resistivity is 1000 Ohmm.
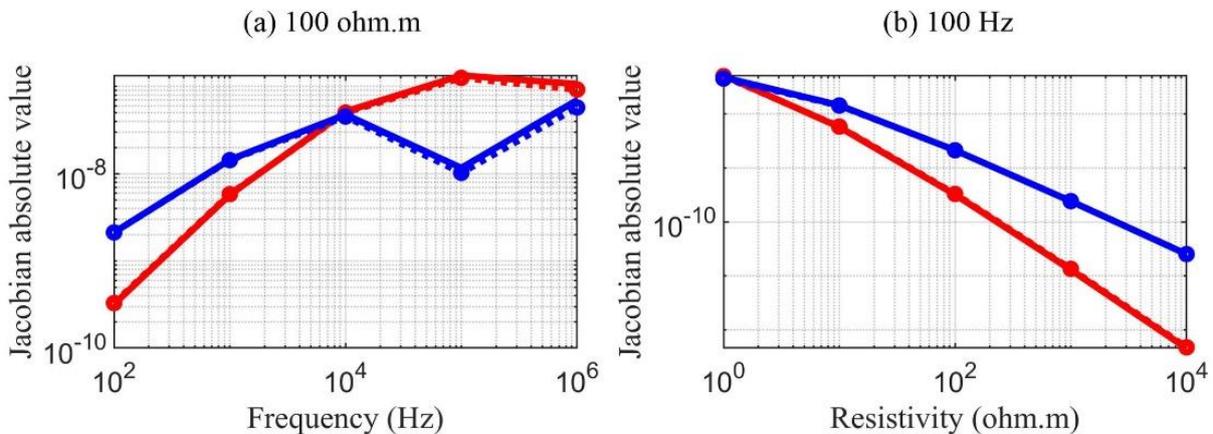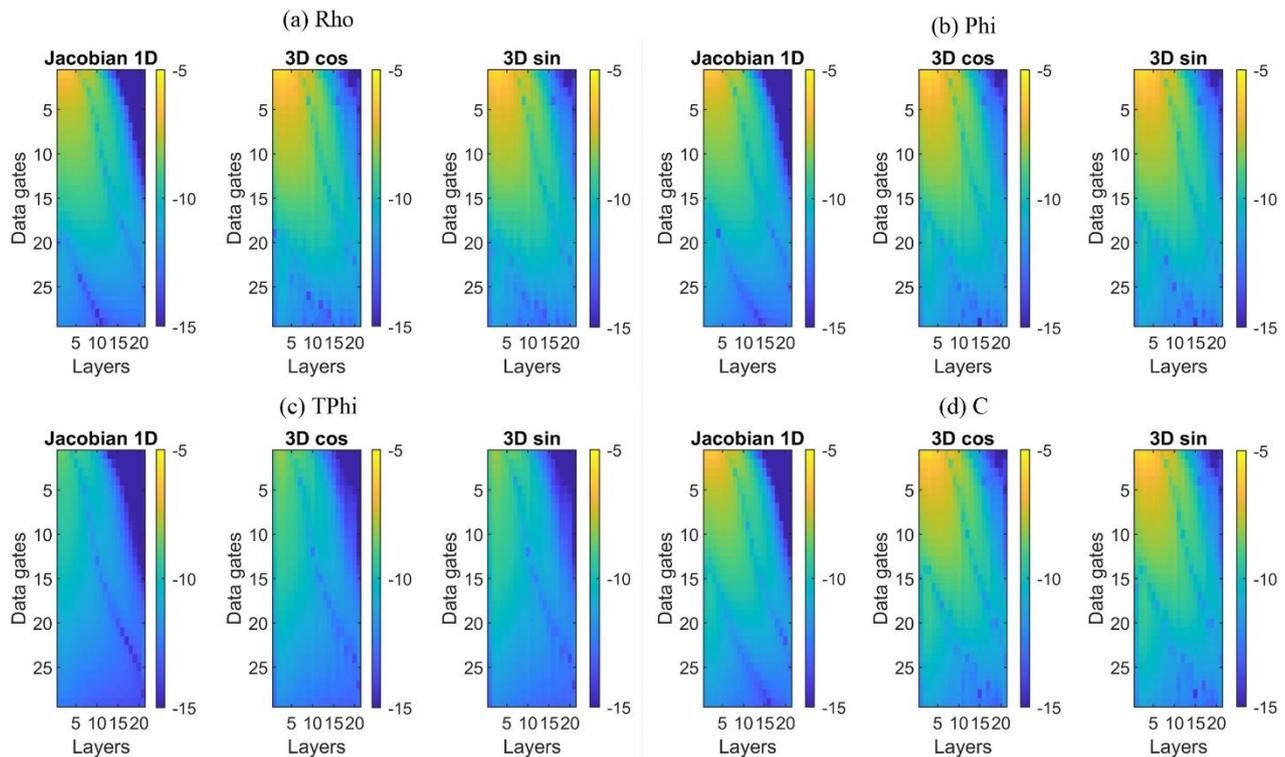

Figure 3. Comparison of calculation accuracy of the Jacobian. (a) Comparison of Jacobian values at different frequencies; (b) Comparison of Jacobian values at different resistivities.

Figure 4. 3D time domain electromagnetic Jacobian matrix and response comparison considering induced polarization effect. MPA re-parameterisations of the Cole–Cole model provided by the EEMverter (Fiandaca et al., 2024) platform. The resistivity value is set to 100 Ohm.m, the Phi value is set to 200, the Tphi value is set to 0.01, and the C value is 0.5.

CONCLUSIONS

To tackle the precision and efficiency issues in the 3D inversion of AEM data, we implemented the vector FE method for forward calculations and employed the Octree mesh to establish local forward mesh. Utilizing the EEMverter modeling platform, we simulated the IP effects in electromagnetic response data using the MPA parameterization model. Through both simulation and field data cases, we validated the accuracy of the 3D forward modeling and demonstrated the significance of 3D inversion for accurate interpretation of mineral deposit distribution.

ACKNOWLEDGMENTS

REFERENCES

Fiandaca, G., Chen, J., Zhang, B., 2024. Closing the gap between galvanic and inductive induced polarization: EEMverter, a new modelling tool for Electric and Electromagnetic data. 1st ASEG DISCOVER Symposium 2024, 15-18 October 2024, Hobart, Tasmania.

Finn, C. A., Bedrosian, P. A., Holbrook, W. S., Auken, E., Bloss, B. R., Crosbie, J., 2022. Geophysical imaging of the Yellowstone hydrothermal plumbing system. Nature, 603, 643–647.

Spitzer, K., 2024. Electromagnetic Modeling Using Adaptive Grids – Error Estimation and Geometry Representation. Surveys in Geophysics, 45, 277-314.

Yoshimura, R. and Oshiman, N., 2002. Edge-based finite element approach to the simulation of geoelectromagnetic induction in a 3-D sphere. Geophysical Research Letters, 29(3),1039.

Zhang, B., Yin, C. C., Liu, Y. H., Ren, X. Y., Baranwal, V. C. and Xiong, B., 2021. 3D inversion of large-scale frequency-domain airborne electromagnetic data using unstructured local mesh. Geophysics, 86(5), 333-342.