Inversion of galvanic time-domain IP data in terms of Debye-Warburg decomposition

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<u>Workshop session:</u> From field data acquisition and processing to inversion
<u>Is the first and presenting author a student?</u> No

The induced polarization (IP) method yields images of the complex resistivity (CR) of the subsurface, which provides information about the conduction and polarization properties of the measured soils or rocks. The measurements can be performed at different frequencies (typically below 10 kHz), in the so-called spectral IP (SIP) method, to obtain information about the frequency dependence of the CR. SIP data are usually analysed and related to petro-hydro, or biogeophysical properties by means of empirical or mechanistic models. Two types of empirical model approaches can be distinguished: The first type describes SIP data using one or a few polarisation peaks (i.e., local maxima) in the absolute phase spectrum (or imaginary conductivity spectrum) with corresponding, distinct relaxation times. This includes the Debye model (Debye, 1960) and the class of Cole-Cole-type models (Cole and Cole, 1941; Pelton *et al.*, 1978; Dias, 2000). The second approach describes the SIP response using a linear superposition of a large number of elementary Debye polarisation terms following a given distribution of relaxation times. This procedure of determining a relaxation time distribution (RTD) instead of a fixed number of relaxation times is referred to as Debye decomposition (DD) (Nordsiek and Weller, 2008). With this approach, the observed CR is represented as a superposition of a large number of Debye relaxation terms at relaxation times τ_k:

$$\rho^*(\omega) = \rho_0 \left(1 - \sum_{K=1}^{N} m_k \left[1 - \frac{1}{1 + (j\omega\tau_k)^c} \right] \right) \quad (1)$$

where N is the number of relaxation times (i.e., Debye polarization terms) used for the superposition, ρ_0 is the DC resistivity, m_k is the k-th chargeability corresponding to the k-th relaxation time. The frequency dispersion of the kernel functions in the decompositions is controlled by the chosen fixed value for c, with the DD resulting for $c=1$ and a Warburg decomposition (Revil *et al.*, 2014) resulting for $c=0.5$

In this scenario, EEMverter (Fiandaca et al., 2024), a novel software to model IP in in electric and electromagnetic (EM) data, allows a parametric definition of electrical properties, such that the (complex) electrical conductivity/resistivity through functions, also integrating petrophysical relations. In particular, the Debye-Warburg decomposition parametrization is also available for inversion of field data, with selectable number of relaxation peaks per frequency decade: typically, one peak per frequency decade is enough for inverting field data.

A synthetic example mimicking unconsolidated sediments in presented in Figure 1. Following Römhild *et al.*, (2022), the forward response was computed by using the WhyCDF model space,

$$\boldsymbol{m}_{WhyCDF} = \{\sigma_w, K, D_+, F\}$$

where σ_w and K are water and hydraulic conductivity, respectively, D_+ is the diffusion coefficient in the Stern layer and F is the formation factor. Full-decay time-domain IP are modelled with a multiple-gradient sequence, and the inversion is performed with a Warburg model space with three relaxation times at 10^{-2}, 10^{-1} and 10^0 seconds:

$$\boldsymbol{m}_{Warburg} = \{\rho_0, m_1, m_2, m_3..\}$$

where ρ_0 is the DC resistivity and m_1, m_2, m_3 are the chargeabilities.

Figure 2 presents the inversion results of the Warburg decomposition, showing a good accordance with the spectral simulated spectral properties. We believe that this new inversion scheme will help in comparing laboratory and field results, offering a modelling framework that allow a dirct comparison of the results.
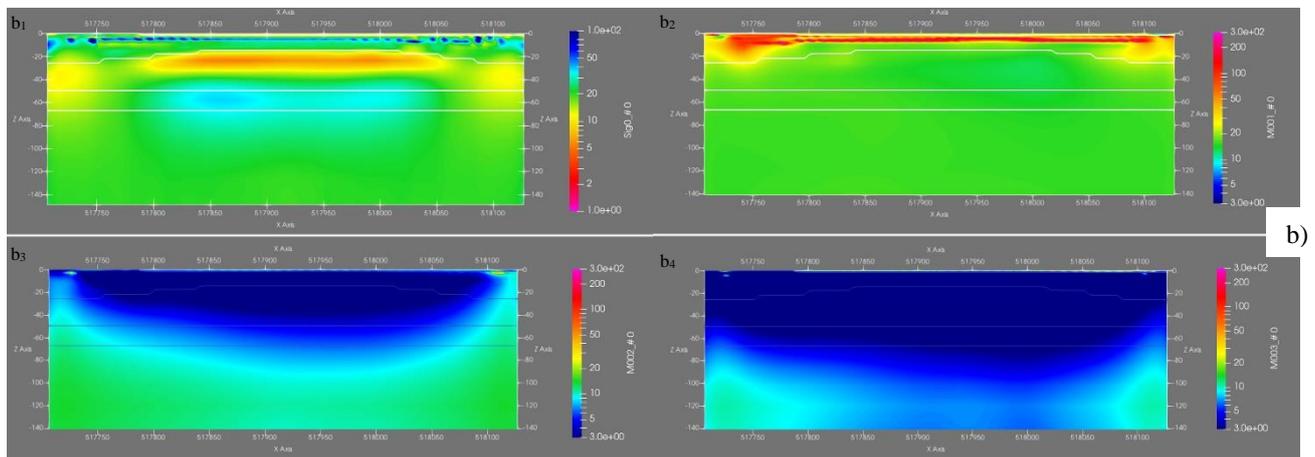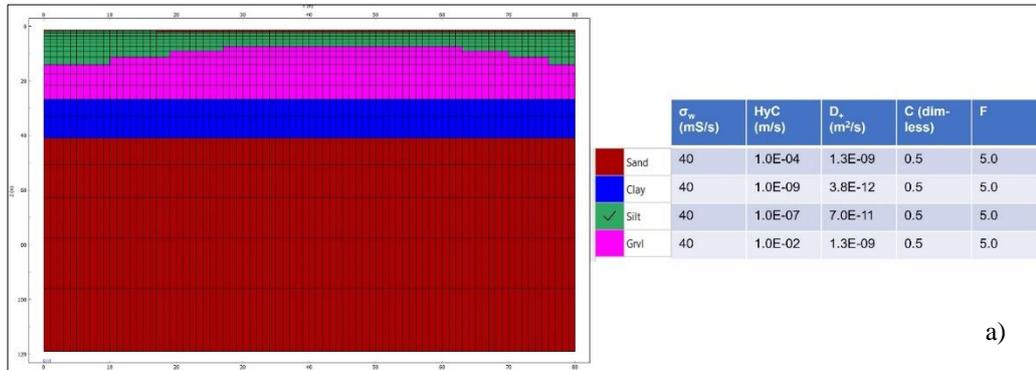

Figure 1:Inversion results of synthetic data. (a): The synthetic model built by using EEMstudio with four different lithologies; b_1,b_2,b_3,b_4: inversion parameters used in the Debye-Warburg decomposition re-parametrization (conductivity, m_1,m_2,m_3, respectively).

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